

## Optimum Adaptive Beamforming Techniques

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### ABSTRACT

Smart antennas possess the capability of suppressing jamming signal, so they can improve signal to interference plus noise ratio (SINR). Array processing utilizes information regarding locations of signal to aid in interference suppression and signal enhancement and is considered promising technology for anti-jamming. In this paper we studied three beam forming algorithms, Least Mean Square (LMS) algorithm, Optimized-LMS algorithm and Recursive Least Squares (RLS) algorithm. Simulation results are presented to compare the ability of these three algorithms to form beam in the direction of desired signal and place null in the direction of interference signal. Dependency of these algorithms on SNR and SIR is also analyzed. It has been found that RLS algorithm is best suited for anti-jamming applications.

Keywords: SINR, SIR, RLS, LMS

### 1. INTRODUCTION

Potential jamming in military and critical civilian applications has been a major concern for system designers. And usual filtering techniques are not helpful as the jamming signal and desired signal are of same frequency. Various methods have been adopted to avoid jamming; including frequency hopping but it requires excessive bandwidth. Spatial filtering can solve the problem [10] over head without the need of additional bandwidth as signals are filtered on basis of their direction of arrival.

Non-blind algorithms as discussed in this paper require the information of desired signal but blind algorithms such as Constant Modulus Algorithm (CMA) and MUSIC algorithm can estimate the Direction of Arrival (DOA) of the source signal, and then this direction information can be utilized in non-blind beam forming algorithms to form beam in the estimated direction. LMS algorithm is known for its simplicity and robustness. The computation complexity of LMS algorithm is  $O(M)$ . While it lacks in convergence speed several modifications to the algorithm are proposed including Optimized-LMS [2] Variable Step Size LMS (VSS-LMS) algorithms [6, 7, 1, 3], variable-length LMS algorithm [7], transform

domain algorithms [6], and recently CSLMS algorithm [9]. Optimized-LMS algorithm modifies the conventional LMS algorithm with optimized step size and is studied in detail in this paper. The computational complexity of Optimized-LMS is also  $O(M)$ . RLS algorithm usually converges with order of magnitude faster than LMS algorithm but the price paid is added complexity. Several variants of RLS algorithm are also proposed one of which is GVFF (Gradient based Variable Forgetting Factor)-RLS [5]. The complexity of RLS algorithm is  $O(M^2)$ .

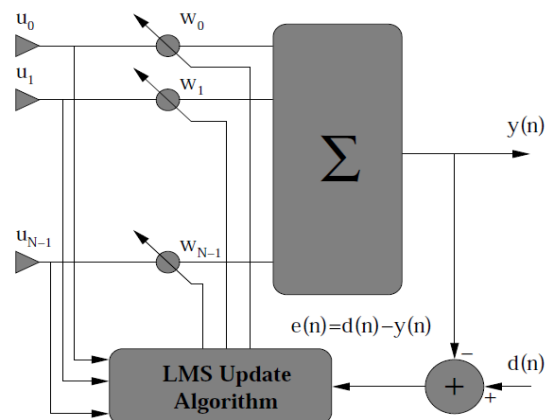


Figure 1. Least Mean Square (LMS) Algorithm

In Fig. 1 the outputs of the individual sensors are linearly combined after being scaled with corresponding weights optimizing the antenna array to have maximum gain in the direction of desired signal and nulls in the direction of interferers.

For beam former the output at any time  $k$ ,  $y(k)$  is given by a linear combination of the data at  $M$  antennas, with  $\mathbf{u}(k)$  being the input vector and  $\mathbf{w}(k)$  being the weight vector

$$y(k) = \mathbf{W}^H(k) \mathbf{u}(k)$$

$$\text{where } \mathbf{W}(k) = \sum_{k=0}^{M-1} \mathbf{w}_k$$

$$\text{and } \mathbf{u}(k) = \sum_{k=0}^{M-1} \mathbf{u}_k$$

The signal received at time index  $k$  is

$$\begin{aligned} r(k) &= u_1(k-1)w_1(m) + \dots + u_l(k-l)w_l(k) + n(k) \\ &= \sum_{j=1}^l u_j(k-j)w_j + n(k) \\ &= \mathbf{u}^T(k) \mathbf{W}(k) + n(k) \end{aligned}$$

The output  $y(k)$  of the adaptive filter is expressed as

$$\begin{aligned} y(k) &= d_1(k-1)w_1(k) + \dots + d_l(k-l)w_l(m) \\ &= \sum_{j=1}^l d_j(k-j)w_j(k) \\ &= \mathbf{D}^T(k) \mathbf{w}(k) \end{aligned}$$

In practice, the adaptive filter can only adjust  $\mathbf{w}(k)$  such that  $y(k)$  closely approximates desired signal over time. Therefore, the instantaneous estimated error signal needed to update the weights of the adaptive filter is

$$\begin{aligned} j(k) &= p(k)e^T(k)e(k) \\ e(k) &= r(k) - y(k) \\ &= r(k) - \mathbf{D}^T(k) \mathbf{w}(k) \end{aligned}$$

This priori error signal,  $e(k)$  is used to minimize the estimator error by adaptive updating of filter weights.

## 2.1 Least Mean Squares (LMS) Algorithm

The LMS algorithm is based on the stochastic gradient and is given by [12]

$$e(k) = \mathbf{u}^T(k) \mathbf{w}(k) + z(k) - \mathbf{u}^T(k) \mathbf{h}(k)$$

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \eta \mathbf{u}(k) e(k)$$

where  $\eta$  is step size,  $\mathbf{u}(k)$  is the transmitted diagonal matrix at sampling time  $k$ ,  $\mathbf{W}(k)$  is the adaptive filter coefficient, and  $e(k)$  is the estimation error. The filter coefficients are updated using an estimate of the cost function gradient,  $[\mathbf{u}(k)e(k)]^T$ . In all practical applications, the signals involved might be corrupted by noise. When the noise is present in the received sequence, interference will also in the coefficients adaption process through the term  $[\mathbf{u}(k)e(k)]^T$ . As a result, where the distribution of the noise is highly impulsive, the LMS scheme might have low convergence and lower steady state MSE performance. The step size parameter,  $\eta$  determines the convergence rate of the algorithm and higher value provides faster convergence. However, if  $\eta$  exceeds certain bound then the algorithm will diverge. As the bound on  $\eta$  is not known a priori and is dependent on the various statistics

## 2.2 Normalized LMS (NLMS) Algorithm

The main problem of the LMS CE algorithm is that it is sensitive to the scaling of its input signals. This makes it very hard to choose  $\eta$  that guarantees stability of the algorithm. The NLMS is a variant of the LMS algorithm that solves this problem by normalizing with the power of the input signal. The NLMS algorithm can be summarized as [11]:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \eta e(k) \frac{\mathbf{u}^T(k) \mathbf{u}(k)}{\|\mathbf{u}(k)\|^2} \mathbf{u}(k)$$

when a constant scalar step size is employed in the LMS are NLMS algorithm, there is a trade off among the steady state error-convergence towards the true channel coefficients, which avoids a fast convergence when the step size is preferred to be small for small output estimation error. In order to guarantee the algorithm to be convergent, the range of step size is specified but the choice of optimal learning step size has not been appropriately addressed. In order to deal with these troubles, one key idea is to exploit varying step size during adaptation.

## 2.3 Recursive Least Squares (RLS) Algorithm

To combat the channel dynamics, the RLS based CE algorithm is frequently used for rapid convergence and improved MSE performance. The standard RLS algorithm is

$$e(k) = \mathbf{u}^T(k)w(k) + z(k) - \mathbf{u}^T(k)w(k)$$

$$R(k) = \lambda^{-1}P(k-1) - \lambda^{-1}R(k)\mathbf{u}^T(k)P(k-1)$$

$$w(k+1) = w(k) + \mathbf{u}(k)e(k)R(k)$$

Where  $\lambda$  is the exponential forgetting factor with  $0 < \lambda < 1$ . The smaller value of  $\lambda$  leads to faster convergence rate as well as larger fluctuations in the weight signal after the initial convergence. On the other hand, too small  $\lambda$  value makes this algorithm unstable. Subsequently, it requires best possible forgetting factor such that the estimator error is decreased.

Although a lot of modified CE algorithm has been studied on employing adaptive forgetting factor and parallel forgetting factor, the CE performance is severely degraded in highly dynamic fading channel even when the forgetting factor is well optimized [13].

## 3. SIMULATION RESULT

A linear 10 element array is simulated in MATLAB to compare results of LMS, Optimized-LMS and RLS algorithm. Spacing between adjacent elements of array equals one half of the wavelength.

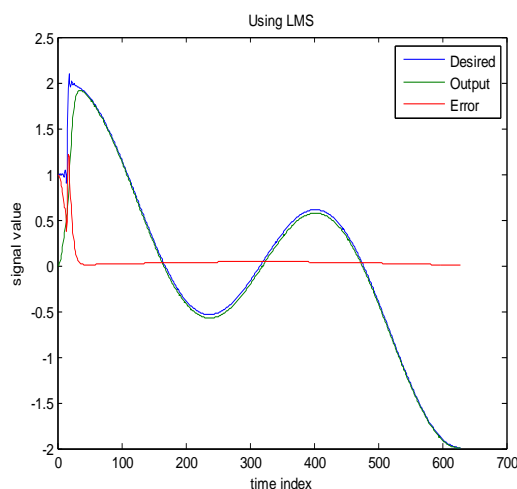


Figure2. Adaptive beam forming using LMS Algorithm

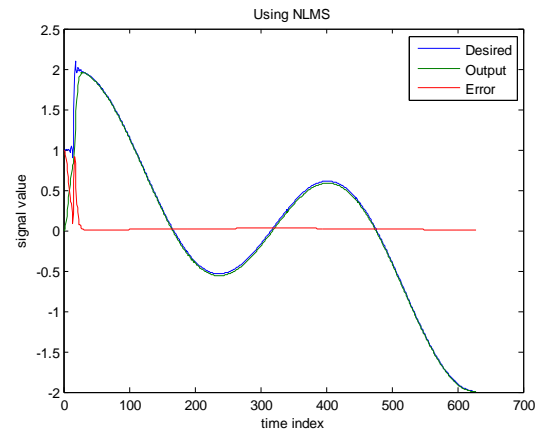


Figure3. Adaptive beam forming using NLMS Algorithm

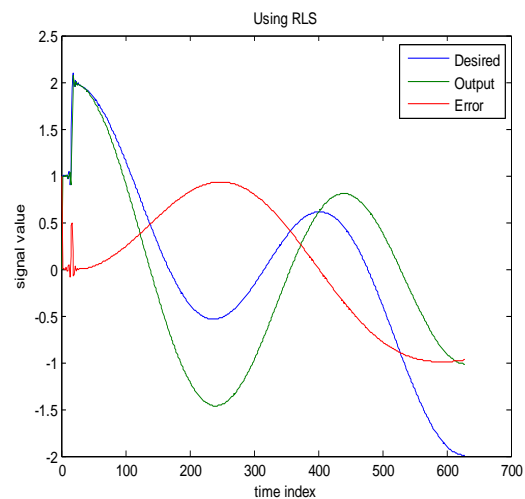


Figure4. Adaptive beam forming using RLS Algorithm

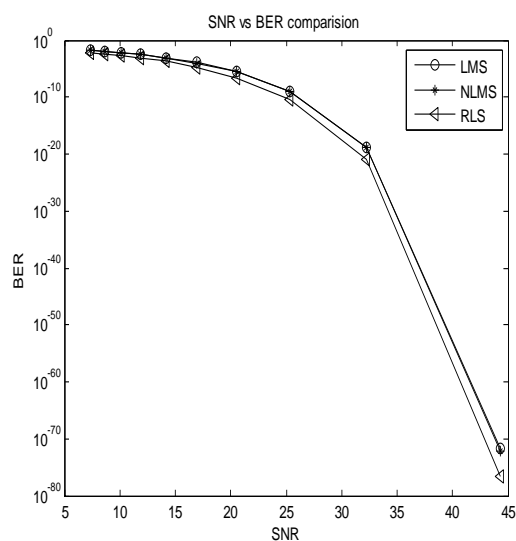


Figure5. Comparison of SNR vs BER for Adaptive beam forming using LMS, NLMS and RLS

#### 4. CONCLUSION

By analyzing the graphs, we observed that RLS algorithm provides fastest convergence, Optimized LMS algorithm also shows fast convergence but LMS algorithm lacks the convergence speed. In beam forming results, RLS showed the best beam forming capability placing deeper nulls in case of all the three interference positions i.e.  $40^\circ$ ,  $60^\circ$  and  $90^\circ$ . The significant difference between the results of LMS and Optimized-LMS in case of beam forming was the presence of many minor lobes in Optimized-LMS. The dependency on SNR and SIR showed that in better conditions i.e., high SNR and SIR Optimized-LMS showed the best results. But in poor conditions i.e. low SNR and SIR its performance deteriorates. LMS and RLS almost showed equal dependency on SNR and SIR. As the recent developments in digital signal processor (DSP) kits and field-programmable gate arrays (FPGA) have made it possible to implement RLS algorithms in real time systems, and complexity to an extent is not a problem anymore. So RLS algorithm is proposed as it provides deeper nulls in the direction of interferences and faster convergence.

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